

## ACADEMIC SCHOLARSHIP 2011

## MATHEMATICS

## PAPER 2

## 2 hours

## CALCULATORS WILL BE NEEDED FOR THIS PAPER.

## INSTRUCTIONS TO CANDIDATES.

You are not expected to have time to do all the questions.
You may answer the questions in any order.
Choose those questions which you think you can answer best.
Remember to show your working and clearly show the method you are using.
Take $\pi$ as either 3.14 or the value on your calculator.
Answers should be given to 3 significant figures where appropriate.
Some questions are longer than others.
The number of marks for each question is shown in square brackets.


1. On $29^{\text {th }}$ April 2011, Prince William married Kate Middleton in a ceremony watched live by 4.5 million people on BBC and ITV. The estimated TV audience around the world was 3 billion, approximately $40 \%$ of the world's population.
a) What does this estimate the population of the world to be?
b) What percentage of TV viewers of the wedding watched on BBC or ITV?
c) A leading supermarket chain reported that they sold an estimated 120 miles of bunting to people celebrating the event. Calculate an estimate of the number of flags this represents, stating any approximations or estimations you use. [Note: 5 miles is approximately the same as 8 kilometres.]
2. My car does 50 miles per gallon. If 1 gallon $=4.546$ litres and the price of petrol is $£ 1.39$ per litre, find the cost in pence of the fuel needed for me to drive from Oundle to Birmingham Symphony Hall, a distance of 74 miles.
3. It takes my Yorkshire friend Dale just 20 hours to shear all his flock of sheep, but last year when Dale was ill, he got his mate Clipper to do the job and he did it in just 15 hours. This year they plan to do the job together. How long should it take them?
4. John goes on a charity journey. He walks a tenth of the way at 3 mph , runs a sixth at 6 mph , cycles a fifth at 12 mph and completes the remaining 16 miles by car at 24 mph . How many miles does he travel and how long does he take?
5. $A B C D$ is a square of side 4 cm . The midpoint of $A B$ is $M$. The diagonal $A C$ meets the line $D M$ at $X$.
a) Give clear reasons why $\triangle C D X$ is similar to $\triangle A M X$.
b) Calculate the area of $\triangle C D X$.
6. The water comes out of my garden hose at 8 metres per second. The internal diameter of the hose is 1.4 cm .
a) Find how much water comes out of the hose in an hour.
b) My swimming pool is 10 metres wide and 20 metres long and slopes steadily from 1 metre deep at the shallow end to 2 metres at the deep end. How long would it take me to fill the swimming pool using the garden hose?
Your answer should be given in days, hours and minutes to the nearest minute
7. Find all the solutions to the equations:
a) $x^{2}=81$
b) $\quad\left(y^{2}-17\right)^{2}=64$
c) $3+\frac{4}{x}=\frac{4}{3+\frac{4}{x}}$
[There is a total of eight solutions to these three equations!]
8. A square $80 \mathrm{~cm} \times 80 \mathrm{~cm}$ has four identical circular discs of radius 20 cm neatly fitting inside it without overlapping, each circle touching two other discs.
a) What percentage of the area of the square is occupied by the 4 discs?
b) A small disc is exactly fitted in the middle so that it touches all four of the discs but does not overlap with any. Find its radius.
9. A circular tunnel has diameter 2 metres and length 100 metres. To enable people to walk along inside the tunnel, a horizontal walkway, of width 1 metre, runs along the length of the pipe (the cross-section is as shown). Find the volume of the pipe which is below the walkway (ignore the thickness of the walkway).

10. I didn't realize there was a power-cut during the night, so I leave to walk to the station as usual, when the electric clock in the hall (which is usually correct) says 8 o'clock. So I am surprised when I get to the station and the clock there says 9.30 , which the ticket inspector assures me is the correct time. I catch the next train, and when I get back in the evening, as I set out for home, the station clock says 7.30 . Of course, I'm tired now, so I can only walk at two-thirds of the pace I managed in the morning. When I get home, the hall clock says 7 o'clock. Assuming there hasn't been another power cut during the day, how long was the power off last night?
11. a) How many 3-digit positive integers can be formed just using the digits 1 to 6 ? [The digits may be repeated, so 244,363 and 111 are all included.]
b) What is the probability that if one of these numbers is chosen at random, it does not include the digit 1 ?
c) If three ordinary dice are rolled together, find the probability of getting a score of at least 16 .
d) If four dice are rolled, find the probability of getting a score of at least 7 .
